Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________

7. $g(-3) =$ ______________

8. $f(t+1) =$ __________

9. $f[g(-2)] =$ __________

10. $g[f(m+2)] =$ ___________

Let $f(x) = \sin x$ **Find each exactly.**

11. $f\left(\dfrac{\pi}{2}\right) =$ ___________

12. $f\left(\dfrac{2\pi}{3}\right) =$ ______________

Let $f(x)=x^2,\ g(x)=2x+5,\ \textit{and}\ \ h(x)=x^2-1$**. Find each.**

13. $h\left[f(-2)\right]=$ _______

14. $f\left[g(x-1)\right]=$ _______

15. $g\left[h(x^3)\right]=$ _______

Find $\dfrac{f(x+h)-f(x)}{h}$ **for the given function *f*.**

16. $f(x)=9x+3$

17. $f(x)=5-2x$

18. $f(x)=2x+1$

<u>Intercepts</u>

Find the *x* and *y* intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Systems

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *and* $x = 5$

Plug $x=3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

Interval Notation

25. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in interval notation.

26. $2x-1\geq 0$

27. $-4\leq 2x-3<4$

28. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

29. $f(x)=x^2-5$

30. $f(x)=-\sqrt{x+3}$

31. $f(x)=3\sin x$

32. $f(x)=\frac{2}{x-1}$

Equation of a line

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: $x = \text{c}$ (slope is undefined)

Horizontal line: $y = \text{c}$ (slope is 0)

33. Use slope-intercept form to find the equation of the line having a slope of 3 and a *y*-intercept of 5.

34. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

35. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

36. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

37. Find the equation of a line passing through the point (2, 8) and normal to the line $y = \frac{5}{6}x - 1$.

38. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

39. Find the equation of a line passing through the points (-3, 6) and (1, 2).

40. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Factor Completely

41. $x^3 + 3x^2 - 10x$

42. $12x^3 - 26x^2 - 10x$

43. $3x^3 - 48x$

Solve for *x*:

44. $4x^2 + 12x + 3 = 0$

45. $2x + 1 = \frac{5}{x+2}$

46. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Solve for the indicated variable:

47. $A = P + xrP,\ for\ P$

48. $2x - 2yd = y + xd,\ for\ d$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

49. $f(x) = \frac{1}{x^2}$

50. $f(x) = \frac{x^2}{x^2 - 4}$

51. $f(x) = \frac{2 + x}{x^2(1 - x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

52. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

53. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

54. $f(x) = \frac{4x^5}{x^2 - 7}$

Unit Circle

55. Evaluate:

a. $\sin\frac{2\pi}{3}$ b. $\cos\frac{7\pi}{4}$ c. $\tan\frac{5\pi}{6}$ d. $\sin 6\pi$

e. $\csc\frac{5\pi}{4}$ f. $\sec\frac{4\pi}{3}$ g. $\cot\frac{\pi}{3}$ h. $\tan\frac{3\pi}{2}$

i. $\sec\frac{3\pi}{4}$ j. $\sin\frac{\pi}{2}$ k. $\cos\frac{8\pi}{3}$ l. $\tan 3\pi$

56. Evaluate in radians:

a. $y=\arcsin\frac{-\sqrt{3}}{2}$ b. $y=\arccos(-1)$ c. $y=\arctan(-1)$

d. $y=\sec^{-1}1$ e. $y=\sec^{-1}\left(\frac{1}{2}\right)$ f. $y=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

g. $y=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ h. $y=\cot^{-1}(-1)$ i. $y=\csc^{-1}\sqrt{2}$

Trigonometric Equations:

Solve each of the equations in the interval $0 \le x < 2\pi$.

57. $\sin x = -\frac{1}{2}$

58. $2\cos x = \sqrt{3}$

59. $\sin^2 x = \frac{1}{2}$

60. $4\cos^2 x - 3 = 0$

61. $\sin^2 x = \sin x$

62. $2\cos^2 x - 1 - \cos x = 0$

Laws of Exponents

Example:		
$x^a x^b = x^{a+b}$	$\frac{x^m}{x^n} = x^{m-n}$	$\left(x^y\right)^z = x^{yz}$

Write each of the following expressions with positive exponents.

63. $\frac{(2a)^3}{b}$

64. $\sqrt{9ab^3}$

65. $\frac{a^{-2}b^4c^0}{a^{-7}bc^5}$

66. $\frac{a^{\frac{1}{3}}a^{\frac{2}{5}}}{a^{\frac{3}{4}}}$

Laws of Logarithms

Example:

$\log_a x + \log_a y = \log_a(x \cdot y)$ $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$ $\log_a x^n = n\log_a x$

$\log_e x = \ln x$ $e^{\ln m} = m$ $\ln e^m = m$

Simplify each of the following:

67. $\log_2 5 + \log_2 10$
68. $2\log_2 9 - \log_2 3$
69. $e^{\ln 5}$
70. $\log\sqrt{10}$
71. $\ln\left(e^{-x}\right)$

Solving Exponential and Logarithmic Equations

Solve for x.

72. $5^{x+1} = 25$
73. $\frac{1}{3} = 3^{2x+2}$
74. $\ln(5x+4) = \ln(3x+18)$
75. $\ln(x+7) = 4$
76. $\log_2 x^2 = 3$
77. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Solve for a.

78. $b^a = 36$
79. $\ln(4a-3) = b$
80. $a^3 = 8b^{10}$
81. $\ln 3a - \ln 5b = 12$
82. $be^a = 7$
83. $e^{3a+5} = b$